Grade 7/8 Math Circles

October 18/19/20/24, 2022

Radians Problem Set

1. Convert the following to radians.

   a) 1° b) π° c) 15° d) 30° e) 120° f) 248° g) 613° h) 840°

2. Convert the following to degrees.

   a) 1 rad b) $\frac{\pi}{2}$ rad c) $\frac{2\pi}{5}$ rad d) 26 rad e) 4π rad f) $\frac{29\pi}{11}$ rad g) $\frac{\pi^2}{2}$ rad

3. To convert from degrees to radians, we multiply an angle measured in degrees by $\frac{\pi \text{ rad}}{180^\circ}$ to cancel out the degree units. To convert from radians to degrees, we multiply an angle measured in radians by $\frac{180^\circ}{\pi \text{ rad}}$ to cancel out the radian units. In both cases, we are multiplying by one since $180^\circ = \pi$ rad.

   Using the idea of multiplying by one and cancelling out the given units, find the factor by which we multiply an angle measured in gradians to convert into degrees and the factor by which we multiply an angle measured in gradians to convert into radians.

4. Calculate the circumference of a circle whose radius is 17 mm.

5. Calculate the area of a circle whose diameter is 6 cm.

6. Calculate the arc length and sector area of sectors with the following central angles and radii.

   a) radius = 2 mm, central angle = 6 rad
   b) radius = 3 cm, central angle = $\frac{\pi}{11}$ rad
   c) radius = 7 mm, central angle = 0 rad
   d) radius = 1 m, central angle = $\frac{87\pi}{46}$ rad

7. Consider a circle whose radius is 3 mm that contains a sector with a central angle of $\frac{10\pi}{11}$ rad. Calculate the arc length of the **major sector**.

8. Consider a circle whose radius is 5 mm that contains a sector with a central angle of $\frac{7\pi}{4}$ rad. Calculate the sector area of the **minor sector**.

9. Consider a circle whose radius is 7 cm. Calculate the perimeter of the sector with a central angle of $\frac{3\pi}{2}$ rad.

   Hint: the perimeter of the sector is more than just the arc length. Review the definition of a sector from the lesson.

10. In the lesson, we rearranged the formula $\theta = \frac{a}{r}$ to get the arc length formula $a = r \times \theta$.

    We can also rearrange the sector area formula $A_S = \frac{1}{2}r^2\theta$. For example, suppose we know the sector area and the radius. If we want to calculate the central angle of the sector, we can rearrange the equation to isolate θ by doing the same operations on both sides of the equation.

    - First, multiply both sides of the equation by 2: $2A_S = r^2\theta$
    - Then, divide both sides of the equation by r^2: $\frac{2A_S}{r^2} = \theta$

    So, we get the following formula for calculating the central angle: $\theta = \frac{2A_S}{r^2}$.

    Rearrange the sector area formula in the same way but isolate the radius. That is, we know the sector area and the central angle but want to calculate the radius.